# MAE140 Linear Circuits Quiz # 1 --- January 29, 2002

## Instructions

This quiz is open book. You may use whatever written materials you choose including your class notes and the textbook. You may use whatever calculator you desire, provided it has no messaging communications capability, including infrared, radio and other wireless technology. These permissions should be taken to indicate the limited help that either written material or computational assistance is likely to provide --- please do not spend significant time looking up books or calculating. That is not what is being tested here. Marks are awarded for concepts and methods.

You should attempt to answer all of the first three questions. They are equal value, although not necessarily equal difficulty.

Question 4 is a bonus question.

You have 70 minutes. I do not expect many people to finish.

Please mark your papers with your name and student number.

## Question 1 --- Circuit Concepts

(a) [50%] Incandescent light bulbs are standard in domestic lighting and use a hot filament to produce light. Current passing through the resistive filament provides the heat. These are your regular light bulbs. A domestic 100W incandescent light bulb designed to operate in USA on the 110V mains supply has a resistance of about 60Ω.
   1. What resistance would a 50W light bulb for USA have?
   2. What resistance would 100W light bulb have if it were designed to operate in Europe on the 220V supply?

(b) [50%] The circuit below in Figure 1 shows a resistor in parallel with both an independent current source and an independent voltage source. Analyze the power flow in the circuit to show that, no matter what the values of $R$, $i_S$ and $v_S$,
   1. the resistor always absorbs the same amount of power,
   2. the current source always provides the same amount of power,
   3. the voltage source either provides or absorbs power to balance the energy flow.

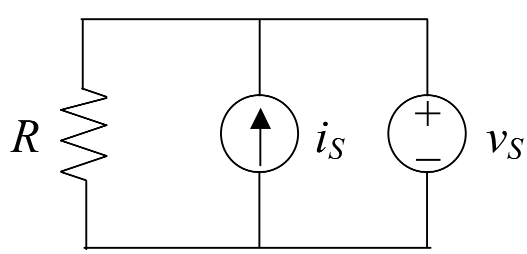



Figure 1

## Question 2 --- Circuit Analysis

Consider the circuit depicted in Figure 2. Write down

1. [45%] The equations from nodal analysis in matrix form,
2. [45%] The equations from mesh analysis in matrix form,
3. [10%] Equations for circuit variables $i_x$ and $v_y$ in terms of the nodal variables and in terms of the mesh variables.

Do NOT solve the equations, unless you really feel the need. There are no marks for that. (I get $i_x$= 2.3256A and $v_y$=-30.5426V.)

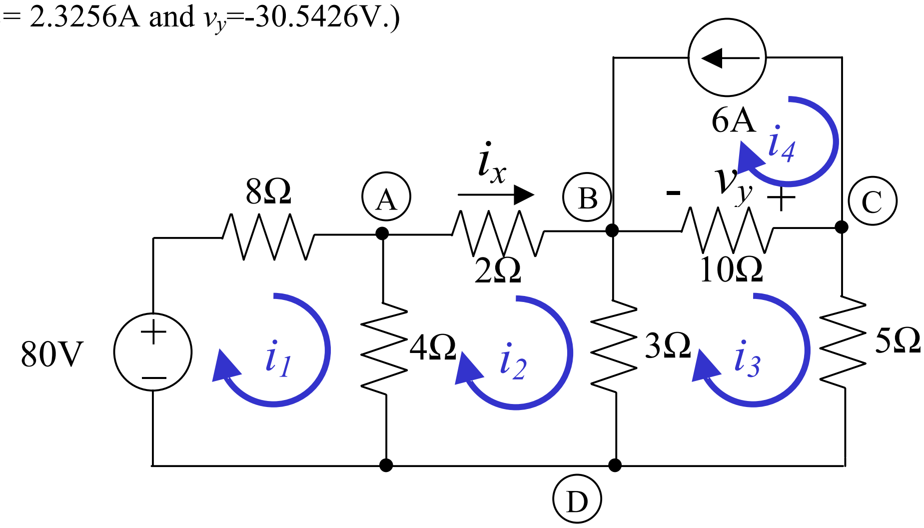



Figure 2

## Question 3 --- Circuit Analysis

Consider the circuit in Figure 3 and, using the method of your choice, write down the equations in matrix form suitable for entry into Matlab to permit the solution for voltage $v_0$. Show your working. Do NOT solve the equations unless you really want to – there are no extra marks for that. (I get $v_0$=8V.)

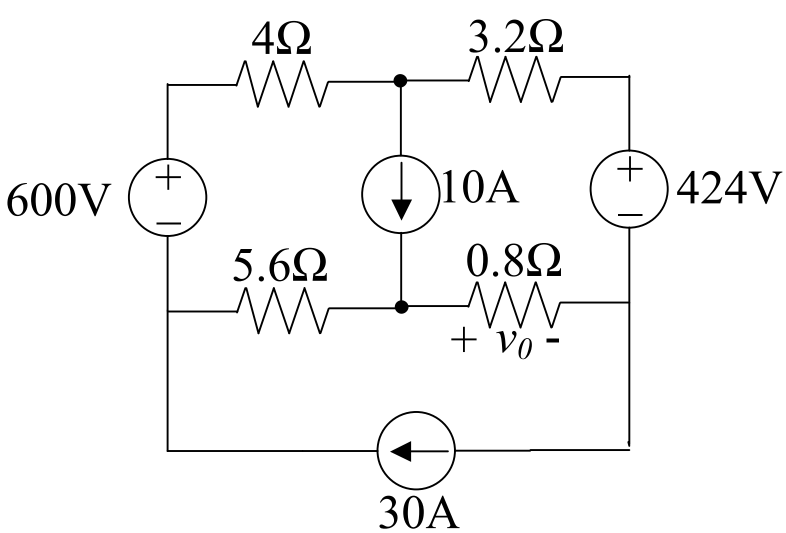



Figure 3

## Bonus Question 4 – Multiport Concepts

Consider the circuit in Figure 4 below.

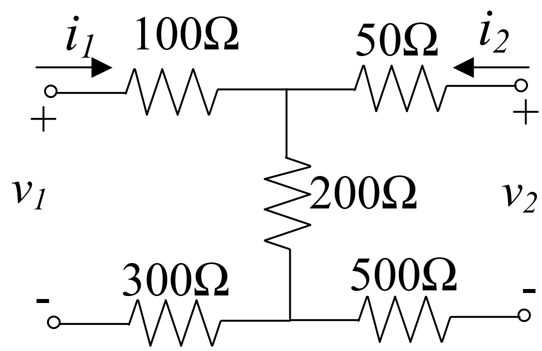
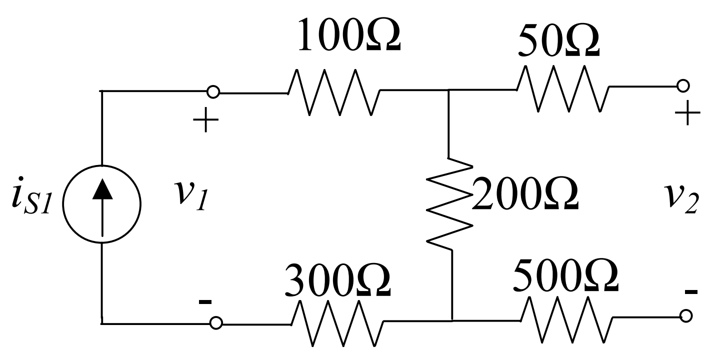



Figure 4. Two-port circuit

This circuit has two "ports" or terminal pairs. The left pair has port variables voltage $v_1$ and current $i_1$ assigned with directions as shown. Similarly for the illustrated right port variables $v_2$ and $i_2$.

1. [15%] Connect a current source at port one to drive current $i_{S1}$ amps into the positive terminal of $v_1$ while keeping $i_2$ zero as shown below. Compute the response values of the two voltages $v_1$ and $v_2$.




2. [15%] Repeat the calculation for the two voltage responses to a current source $i_{S2}$ amps connected to drive current into the positive $v_2$ terminal while port one is kept open circuit.
3. [15%] Use these two calculations to express voltage responses $v_1$ and $v_2$ as a linear combination of driving currents $i_1$ and $i_2$, where each of these currents can take on an arbitrary value. Write this expression in matrix form $\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix}$ with numerical values for the $r_{ij}$.
4. [15%] We now introduce the concept of a "positive definite" matrix. We say an $n \times n$ real matrix, $Z$, is positive definite if it is symmetric and if for any non-zero $n$-vector $x$ we have $x^T Z x > 0$.

A test for positive definiteness is that all the "leading principal minors" of $Z$ are positive. The leading principal minors of an $n \times n$ matrix are the $n$ determinants created by taking the first $k$ rows and columns of $Z$ for $k=1,2,\ldots,n$. For our $2\times2$ matrix above, there are two leading principal minors, $r_{11}$ and $(r_{11}r_{22}-r_{12}r_{21})$.

Show that your calculated matrix is positive definite.

5. [40%] Consider the product $\begin{pmatrix} i_1 & i_2 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} i_1 & i_2 \end{pmatrix}\begin{pmatrix} r_{11} & r_{12} \\ r_{12} & r_{22} \end{pmatrix}\begin{pmatrix} i_1 \\ i_2 \end{pmatrix}$,

   which is now of the form $x^T Z x$.

   Interpret the physical meaning of this matrix $Z$ of resistances being positive definite.