Controller Certification

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***Abstract*— In experimentally assessing the stability and performance level of a large collection of controllers in feedback with a single unknown actual plant, we develop a method of searching for a subset of controllers to manage the number of the experimental assessments. Avoiding testing every prospective controller, we check whether every controller in the large set of controllers will work satisfactorily when they are connected with an actual but unknown plant by experimenting only a small subset of candidate controllers. In doing this, the ν-gap metric will be a main tool.**

I. INTRODUCTION

The problem of large-scale controller validation via experimental proof of stability and performance that is provided by one of colleague controllers is studied. If there are a large number of controllers, for cost and time reasons it would be unrealistic to evaluate them all by experiments. We look for an effective way to check whether a large set of model-based controllers will work satisfactorily when they are connected with an unknown actual plant. Since the designed controllers are based on the plant model, which represents only part of real physics of an actual plant, stability and performance of candidate controllers are needed to be corroborated by closed-loop experimental data before commissioning them for application. Furthermore if the controllers are used in a system that requires a high degree of safety and performance, then the experimental validation of controllers becomes an indispensable task.

Recently this safety issue was studied by the Action Group 11 of the GARTEUR(Group for Aeronautical Research and Technology in Europe) who made a research effort to develop advanced clearance techniques proving that the flight control system is safe and reliable and has the desired performance under all possible flight conditions and in the presence of controller failures before an aircraft can be tested in flight [1]. In their research, using advanced mathematical analysis tools including the ν-gap metric, they explored the potential improvement of clearance procedures. In that book, Steele and Vinnicombe used the ν-gap metric to obtain a linearized approximate model of a nonlinear system with parametric uncertainty. The industrial clearance process for flight control laws is an extensive task because of many constraints and varying parameters which influence stability and handling of an aircraft. A controller is said to be certified if it can be guaranteed to stabilize the actual plant with a prescribed performance level. We are looking for a small subset of controllers to test by physical experiments so that we can manage the number of experiments. Experimental tests on this small number of controllers guarantee that the required degree of stability and performance will be attained by all certified candidate controllers when they are in the actual closed-loop.

The importance of using experimental evaluation of designed controllers is advocated by controller unfalsification [2]. By using experimental input-output data of a plant, the unfalsified control theory sifts the controllers that are demonstrably unrobust from a set of candidate controllers. We also make use of experimental data to group the designed controllers into subset of satisfactory controllers and rejected controllers. However, in this paper, we guarantee certified controllers to have required degree of stability and performance. Controller certification is the question of which controllers should be chosen for experimental evaluations. The quantitative feedback theory (QFT) [3] has been proposed as a computational approach to deal with a related problem of determining whether a controller is capable of stabilizing a number of different plants. Here the roles of plant and controller are reversed, but the method otherwise is related. The tack taken here is ideologically similar to QFT, which is based typically on Nichols charts and frequency response data. However we use analytical tools such as the frequency-dependent Vinnicombe ν-gap metric as a more formal tool which deals rationally and computationally with the MIMO nature of the problem.

The certification problem arises in jet engine control for a Short Takeoff and Vertical Landing (STOVL) aircraft. Constraints are connected with the engine hardware limits such as actuator amplitude/rate limits, operating temperature limits, cooling flow pressure ratio limits, and compressor stall margin limits to operation. The upper limits will not be active at the same time with lower limits on the one variable. The engines are necessarily highly coupled MIMO systems. The same engine operates in low altitude, low velocity, high power demand for vertical takeoff and in supersonic flight, leading to dramatically different constraint sets

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operating. For example if there were 20 constraints and four constraints of them were active at any one instance, there are 4845 possible combinations of constraints. According to the change of active constraints, the controller may modify its parameters and this modification makes a finite but large number of candidate controllers. The plant model also must be adjusted for operating points. However, this latter variation is not yet our concern, nor do we consider time-varying stability due to controller change or operating point change since this requires at least stability each stationary value. In practice, the plant-controller testing will need to be fully MIMO and to permit the computation of generalized stability margin, $b_{P,C}$ (to be introduced later), as a function of frequency. This will require $\mathbb{H}_\infty$-norm estimation, see [4], witch is data-intensive experimental test for each controller. This motivates our desire to minimized the numbers of necessary test points.

It is assumed that we have a fixed but unknown linear actual plant, P, and known linear model, $\hat{P}$. We also presume that each controller in the given set of candidate controllers, $\mathscr{C} = \{C_0, C_1, C_2, \cdots\}$, either finite or infinite, has been designed to stabilize the nominal model, $\hat{P}$ with a specified performance level. The problem is to select a subset of controllers from $\mathscr{C}$ for experimental test with P so that we can separate the complete set into a certified subset and an uncertified or rejected subset. The first category of the problem is certification of a set of finite number of candidate controllers, which is considered in Section 3. In Section 4 an illustrative example of solving the first category is provided. The second class is certification of an infinite number of controllers, which is considered in Section 5. If the controller parameters continuously change in a parameter space, then the set of designed controllers becomes an infinite set. In the next section we provide basic definitions and a theorem for stability and performance guarantees based on the ν-gap metric.

II. Approach

The generalized sensitivity function of the plant-controller feedback pair (P,C) is given by

$$T(P,C) = \begin{pmatrix} P(I+CP)^{-1}C & P(I+CP)^{-1} \\ (I+CP)^{-1}C & (I+CP)^{-1} \end{pmatrix}. \quad (1)$$

Then the generalized stability margin of (P,C) is defined by the generalized sensitivity function as follows

$$b_{P,C} = \begin{cases} (\|T(P,C)\|_\infty)^{-1}, & \text{if } (P,C) \text{ is stable} \\ 0, & \text{else.} \end{cases} \quad (2)$$

McFarlane and Glover [5] used $b_{P,C}$ to denote a neighborhood of perturbations about the normalized coprime factors of P stabilized by C such that the perturbed closed-loop system will remain stable. The fact that $b_{P,C} > 0$ guarantees the feedback pair (P,C) is stable and a higher value of $b_{P,C}$ equivalent to a better degree of stability. As well as being a stability measurement, $b_{P,C}$ is quantification of performance level. Performance can be defined in different methods and a single performance criterion might not be entirely applicable to every situation. Intuitively, however, the performance of (P,C) will be severely degraded with a very small $b_{P,C}$ in the sense that (P,C) will goes to instability by small amount of perturbations. This fundamental value $b_{P,C}$ can be retrieved from experimental data.

Definition 1 (Certification): Given a set of controllers, not necessarily finite, $\mathscr{C}$, which is designed to stabilize the plant model $\hat{P}$, a controller C_j is said to be *certified* if, using experimental data with the unknown actual plant P, we can guarantee that the generalized stability margin of the pair (P,C_j) is greater than a pre-specified performance level $\alpha \in [0,1)$,

$$b_{P,C_j} > \alpha. \quad (3)$$

Otherwise, if we can prove that $b_{P,C_j} \leq \alpha$ then C_j is said to be *rejected*.

Clearly, one could certify or reject all candidate controllers in a finite set by testing experimentally all pairs (P,C_j). Our aim here is to devise a systematic procedure for testing only a restricted subset of the candidate controllers and thence certifying all the remaining controllers. For a infinite controller set, one must rely on certification based on tests involving finite subsets of the candidates. The central tool that we shall apply in controller certification is the Vinnicombe ν-gap metric[6], $\delta_\nu(\cdot,\cdot)$, which measures the distance between two systems or controllers yielding a number $\delta_\nu(C_i,C_j) \in [0,1]$. The key relation of the ν-gap metric follows.

Theorem 1 (Vinnicombe): Consider a plant P and two controllers C_i and C_j, with C_i stabilizing P. Then the following results hold.

Stability guarantee: (P,C_j) is stable if

$$\delta_\nu(C_i,C_j) < b_{P,C_i}. \quad (4)$$

Performance guarantee: If $\delta_\nu(C_i,C_j) < b_{P,C_i}$ then

$$\arcsin b_{P,C_j} \geq \arcsin b_{P,C_i} - \arcsin \delta_\nu(C_i,C_j), \quad (5)$$

and further, if C_i and C_j both stabilize P,

$$\delta_\nu(C_i,C_j) \leq \left\| T(P,C_i) - T(P,C_j) \right\|_\infty \leq \frac{\delta_\nu(C_i,C_j)}{b_{P,C_i} b_{P,C_j}}. \quad (6)$$

Notice that Theorem 1 is only a sufficient condition for stability guarantee, i.e., at a ν-distance greater than b_{P,C_i} from C_i, there might exist a controller that stabilizes the plant, P. Further note that equipping the set of controller systems with a metric creates a metric space within which one may define neighborhoods. Using the ν-gap metric and the generalized stability margin, we can define the largest neighborhood of controllers about a given stabilizing controller for which a minimum level of performance (measured by $\|T\|_\infty^{-1}$) is guaranteed with the plant. We have the following immediate result.

Lemma 1: Given b_{P,C_i} from a stable (P,C_i) plant-controller pair, if $\delta_\nu(C_i,C_j)$ satisfies

$$\arcsin\delta_\nu(C_i,C_j) < \arcsin b_{P,C_i} - \arcsin\alpha,$$

then controller C_j satisfies (3).

In Theorem 1, if $\delta_\nu(C_i,C_j) = 1$, each controller cannot guarantee the stability of the other controller, no matter how big their generalized stability margins might be. In this case, it seems that we have to do the experiment twice to find whether both b_{P,C_i} and b_{P,C_j} are sufficiently large. There could exist a controller, C_k, such that $\delta_\nu(C_i,C_k) < 1$ and $\delta_\nu(C_k,C_j) < 1$, since Vinnicobme's winding number is not transitive[7]. Furthermore, if b_{P,C_k} is sufficiently large so that the controller, C_k, is able to satisfy both

$$\begin{aligned}\arcsin\delta_\nu(C_i,C_k) &< \arcsin b_{P,C_k} - \arcsin\alpha\\ \arcsin\delta_\nu(C_k,C_j) &< \arcsin b_{P,C_k} - \arcsin\alpha\end{aligned}$$

then both $b_{P,C_i} > \alpha$ and $b_{P,C_j} > \alpha$ are guaranteed by the Lemma 1. Thus if we choose C_k as a controller on which an experiment is done to find the generalized stability margin, the certification for both C_i and C_j can be finished by a single experiment evaluating b_{P,C_k}. From this fact, we notice that the total number of experiments for solving the certification problem depends on the choice of controller. For a given α, the number of experiments will decrease as the generalized stability margin of the chosen controller, C_k, increases and the number of candidate controllers in the δ_ν-neighborhood of the chosen controller, C_k, increases. In the next section, we will develop an algorithm to search for such a controller C_k by using $b_{\hat{P},C_k}$ in stead of b_{P,C_k}.

III. Finite Controller Set

In this section the candidate controller set has a finite number of elements, i.e. $\mathscr{C} = \{C_0, C_1, \cdots, C_n\}$. To certify a collection of controllers, we need to guarantee the generalized stability margins $b_{P,C_i} > \alpha$ for all $C_i \in \mathscr{C}$. Since the transfer functions of the plant model and all the controllers are known, the ν-gap distances of candidate controller pairs and the $b_{\hat{P},C}$ can be easily calculated. However, the $b_{P,C}$ must be evaluated through an experiment which is a relatively time consuming and expensive process. Instead of b_{P,C_i}, we will let $b_{\hat{P},C_i}$ guide us to determine which subset of controllers should be tested so that we can reduce the number of experiments required to finish certification of the whole set. If a nominal plant model $\hat{P}$ sufficiently approximates a unknown real plant P, the $b_{\hat{P},C_i}$ provides good guidance to solve the certification problem.

Algorithm: Before starting certification for the collection of candidate controllers, $\mathscr{C}$, we need to compute $b_{\hat{P},C_i}$ for all $i = 0, 1, \cdots, n$. Other data we need to prepare are a numerical table comprising the ν-gap distances $\delta_\nu(C_i,C_j)$ for all controller pairs in $\mathscr{C}$. Since the ν-gap is a metric, this table will have a form of $n \times n$ symmetric matrix whose diagonal elements are zeros. An algorithm solving the certification problem is as follows.

Step 1(Searching) For each uncertified controller, C_i, count the number of uncertified controllers, C_j, that satisfy the following,

$$\arcsin\delta_\nu(C_i,C_j) < \arcsin b_{\hat{P},C_i} - \arcsin\alpha, \tag{7}$$

where α is the performance specification. Then choose the controller, C_i, with the most controllers, C_j, satisfying (7).

Step 2(Experiment) Perform the experiment on (P,C_i) to retrieve b_{P,C_i} from close-loop data.

Step 3(Certifying) Certify the controllers, C_j, satisfying the Lemma 1,

$$\arcsin\delta_\nu(C_i,C_j) < \arcsin b_{P,C_i} - \arcsin\alpha, \tag{8}$$

and eliminate these candidate controllers from further tests. That is, for those controllers, C_j, we have shown that $b_{P,C_j} > \alpha$.

If uncertified controllers remain in the collection $\mathscr{C}$, iterate from Step 1 to Step 3 until all controllers are certified or rejected.

Notice that the only difference between (7) and (8) is that pre-computed $b_{\hat{P},C}$ is used in (7), while experimentally retrieved $b_{P,C}$ is used in (8).

IV. Numerical Example

In this section, we will show a computer example of a controller certification procedure using the algorithm developed in the previous section. We consider that controllers are parametrized by two parameters (ρ_1; ρ_2) and presume that the range of parameters that stabilizes the nominal plant $\hat{P}(z)$ is given from a separate controller synthesis process.

$$C(z) = \frac{\rho_1 z}{z+\rho_2} \tag{9}$$

Let us assume, the unknown real plant $P(z)$ is

$$P(z) = \frac{0.1z^2(z-0.3)}{(z-0.8)(z^2-0.4z+0.85)(z^2-1.2z+0.72)}. \tag{10}$$

The stability margin b_{P,C_i} may be estimated from closed-loop data. Although, in this example, because the mathematical expression (10) of the real plant $P(z)$ is available, the experiment estimating the stability margin b_{P,C_i} will be substituted by computation of (2). We obtain the plant model $\hat{P}$ from the model reduction of the real plant $P(z)$.

$$\hat{P}(z) = \frac{0.002985z^2+0.08012z+0.1259}{z^3-1.123z^2+1.014z-0.394} \tag{11}$$

Since $\delta_\nu(P(z),\hat{P}(z))$ = 0.1917 which is an unknown value, the maximum difference between b_{P,C_i} and $b_{\hat{P}C_i}$ is guaranteed

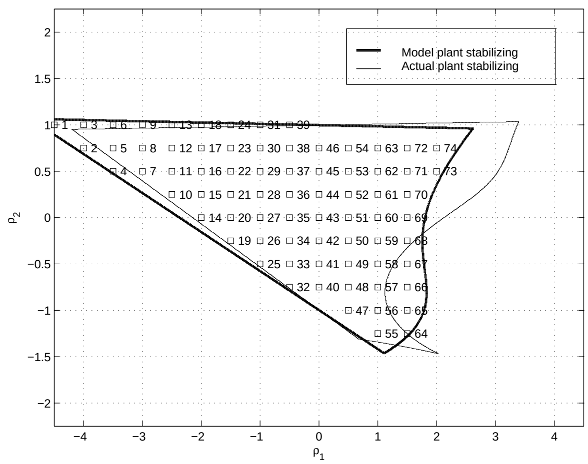

Fig. 1. Stabilizing region for the real plant $P(z)$ and the nominal plant $\hat{P}(z)$ and selected controller parameters.

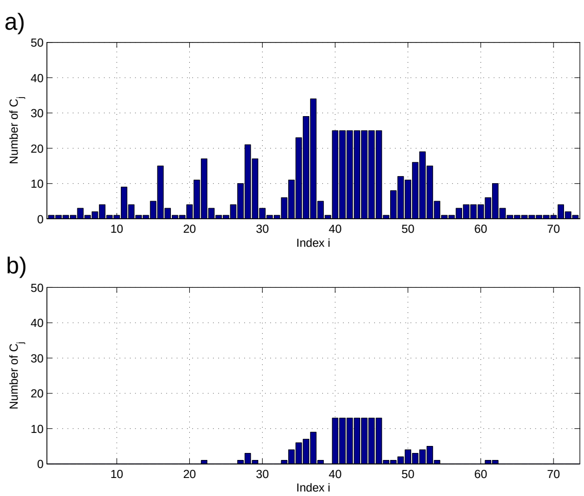

Fig. 2. Numbers of C_j satisfying (7) after 1st searching using $b_{\hat{P},C_i}$, a) $\alpha = 0$, b) $\alpha = 0.3$.

to be less than 0.1917 by Theorem 1. In this example the actual maximum difference between b_{P,C_i} and $b_{\hat{P}C_i}$ is 0.1874.

In Figure 1, the axes measure controller parameters ρ_1 and ρ_2. The thicker line depicts the boundary of the stabilizing region for the model $\hat{P}(z)$ and the thinner line delineates the $P(z)$-stabilizing region. Notice, once again, that we presume the boundary for the $P(z)$-stabilizing region is unknown. Inside thick solid line, we select 74 points of (ρ_1; ρ_2), i.e. 74 model-based controllers, which are represented by rectangles with their indices. As a preparation step we calculate $\delta_\nu(C_i, C_j)$ for all pairs of controllers in $\mathscr{C}$ so that we can use these distances when we check for the certification conditions (7) and (8). Using (2), the generalized stability margins $b_{\hat{P}C_i}$ of all the controllers $\mathscr{C}$ with the nominal plant $\hat{P}$ are calculated in advance.

A. *Certification for Stability*

As a minimum requirement, the candidate controllers should guarantee stability when they are applied to an actual feedback loop. Certification for stability is accomplished by letting $\alpha = 0$ in (7) and (8). The key step in the controller certification procedure is the choice of controller which will be tested first. Hence we start the controller certification by counting the number of controllers satisfying (7) at each candidate. Figure 2 a) shows the number of controllers, C_j, that satisfy (7) at each index $i = 1\cdots,74$. After calculating $b_{\hat{P},C_i}$ we count the controllers C_j. Figure 2 a) shows that $b_{\hat{P},C_j}$ advises us that the 37th controller can certify 34 other controllers from the $\hat{P}$-stabilizing region. After retrieving $b_{P,C_{37}}$ from the closed-loop data, we realize that actually 21 controllers satisfy (8). In Figure 3 a) the controllers inside and on the line are those now certified controllers.

We iterate this search and experiment process until all controllers in the set $\mathscr{C}$ have been certified or rejected. Before searching for the next controller to be tested, we need to exclude the certified controllers in the previous iteration step from $\mathscr{C}$ and search for the best remaining controller for the second experiment. In this manner, as shown in Figure 3, we complete the certification process for all 74 controllers in 31 experiments. In Figure 3, the filled squares correspond to the controllers on which the experiments have been performed. For those controllers inside and on the solid line, the certification criterion (3) is satisfied. From Figure 3 d), we can see that inside and on the line there are 60 certified controllers out of 74 candidate controllers. 14 controllers are rejected.

Since we are exhaustively testing all 74 possible controllers which extend outside the stabilization region for P, and since we are applying a sufficient condition for stability, it is necessary to test experimentally all rejected controllers as well as many near the stabilization boundary. Figure 3 b) indicates that two experiments yield the certification of the bulk of the certifiable controllers.

B. *Certification for Performance*

It is important in controller certification not only to guarantee stability, but also to achieve a specified level of performance. When $\|T(P,C_i) - T(\hat{P},C_i)\|_\infty$ is large, the controller based on the model $\hat{P}$ cannot guarantee good performance with an actual plant P. The dual inequality of (6), with $\delta_\nu(P,\hat{P}) < 1$, is

$$\begin{aligned}\delta_\nu(P,\hat{P}) &\le \left\|T(P,C_i) - T(\hat{P},C_i)\right\|_\infty \\ &\le \frac{\delta_\nu(P,\hat{P})}{b_{P,C_i} b_{\hat{P},C_i}}.\end{aligned} \qquad (12)$$

For any particular $\delta_\nu(P,\hat{P})$, if we find a certain controller that makes $b_{\hat{P},C_i}$ large, then the $\|T(P,C_i) - T(\hat{P},C_i)\|_\infty$ may be smaller, which means we can achieve a better performance level with the chosen controller C_i. By increasing α in (7) and (8), we would expect better performance in the certified

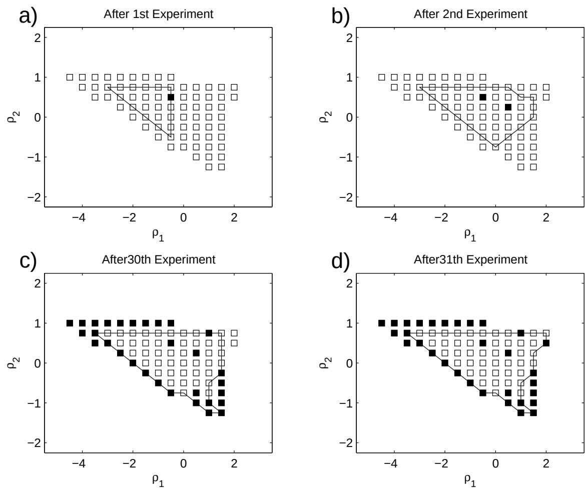

Fig. 3. Certification process for stability ($\alpha = 0$).

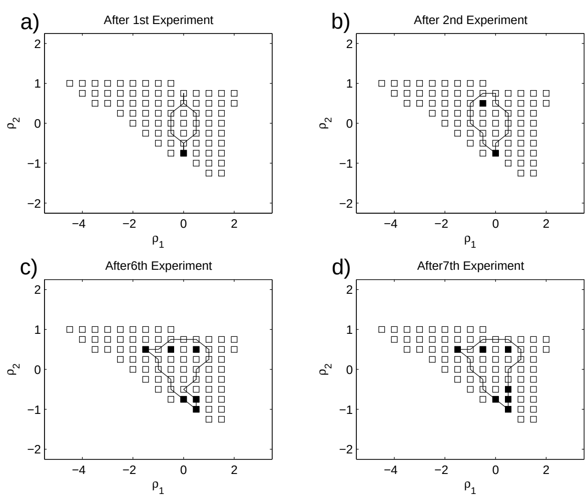

Fig. 4. Certification process for performance ($\alpha = 0.3$).

controllers. Figure 2 b) shows the numbers of controllers satisfying (7) after the first search for the best controller, when $\alpha = 0.3$. This figure tells us that there are 47 controllers that the algorithm will not try to certify since $b_{\hat{P},C} \leq 0.3$ for those controllers. Figure 4 shows the process of certification for 74 controllers with $\alpha = 0.3$ and shows that 27 controllers are certified in 7 experiments by the suggested algorithm.

V. Infinite Controller Set

In the previous section the proposed algorithm certified only a finite number of points in the 2-dimensional continuous parameter space and therefore infinitely many untested controllers are still left. We now move from finite set to an infinite controller set and consider the modification of the preceding enumerative algorithm. The key ideas are the properties of the $(\mathscr{C}, \delta_\nu)$ metric space to develop a sequence of controllers $\{C_0, C_1, \cdots\}$ for experimental test which validate all candidate controllers in an open neighborhood of $\mathscr{C}$. In this section we provide answers to main issues in developing an algorithm which eventually certifies the whole continuous parameter space without testing exhaustively.

Anderson et al [8] establish that for a controller set $\mathscr{C}$ continuously parametrized by parameter θ in a compact set Θ, it is possible to construct a finite open covering of $\mathscr{C}$ by δ_ν-balls about centers $\{C_0, C_1, \cdots\}$. This relies on the Heine-Borel property and pivots on continuity and compactness. However, it is important to note that the controllers must maintain coprimeness when they continuously move from one parameter to another in Θ. If the set of rational functions of a fixed degree n (without common factors) is topologized in a natural way, the set is the disjoint union of $n+1$ open sets [9]. When a controller moves form one of these open sets to another, it must pass through a region of common factors. If there are unstable common factors, the controllers cannot cross one controller set to another without an unstable pole-zero cancellation which causes violation of coprimeness of controllers. Thus the certification algorithm should restrict one individual δ_ν-ball to be contained in only one disjoint set. These disjoint sets contain transfer functions with the same Cauchy index.

We shall apply these ideas here to explore this construction for a specific natural parametrization associated with the controller certification problem. For ease of presentation alone, we restrict ourselves to scalar (SISO) controllers. Assume the candidate controllers are parametrized as follows

$$C(\theta_0) = \frac{b_{0,0} + b_{1,0}z^{-1} + \cdots + b_{n,0}z^{-n}}{1 + a_{1,0}z^{-1} + \cdots + a_{n,0}z^{-n}} \triangleq \frac{n_0(z)}{d_0(z)},$$

$$C(\theta_1) = \frac{b_{0,1} + b_{1,1}z^{-1} + \cdots + b_{n,1}z^{-n}}{1 + a_{1,1}z^{-1} + \cdots + a_{n,1}z^{-n}} \triangleq \frac{n_1(z)}{d_1(z)}.$$

And define an infinite collection of controllers to be certified,

$$\mathscr{C} \triangleq \{C(\theta_i) | \theta_i \in \Theta\}.$$

Where $\theta_i (i = 0, 1, 2, \cdots)$ is a parameter vector such that

$$\theta_i = (a_{1,i}, a_{2,i}, \cdots, a_{n,i}, b_{0,i}, b_{1,i}, \cdots, b_{n,i})^T \in \Theta,$$

and $\Theta \subset \mathbb{R}^{2n+1}$ is a controller parameter space. Suppose the known plant model has normalized coprime factorizations,

$$\hat{P} = XY^{-1} = \tilde{Y}^{-1}\tilde{X} \tag{13}$$

Define unit transfer functions such that

$$U_0(z) \triangleq n_0(z)X + d_0(z)Y,$$
$$U_1(z) \triangleq \tilde{X}n_1(z) + \tilde{Y}d_1(z).$$

Then,

$$I = U_0(z)^{-1}n_0(z)X + U_0(z)^{-1}d_0(z)Y,$$
$$I = \tilde{X}n_1(z)U_1(z)^{-1} + \tilde{Y}d_1(z)U_1(z)^{-1}$$

Choose $\tilde{N_0}$, $\tilde{D_0}$, N_1, and D_1 such that

$$\tilde{N_0} = U_0^{-1}n_0(z), \quad \tilde{D_0} = U_0^{-1}d_0(z),$$
$$N_1 = n_1(z)U_1^{-1}, \quad D_1 = d_1(z)U_1^{-1}.$$

This provides coprime factorizations of $C(\theta_0) = \tilde{D_0}^{-1}\tilde{N}_0$ and $C(\theta_1) = N_1 D_1^{-1} = [N_0 - QY][D_0 + QX]^{-1}$ with Youla-Kucera parameter,

$$\begin{aligned} Q &= \tilde{N}_0 D_1 - \tilde{D_0} N_1 \\ &= U_0^{-1}\left(n_0(z)d_1(z) - d_0(z)n_1(z)\right)U_1^{-1}. \end{aligned} \tag{14}$$

Let us define $\Delta d(z)$ and $\Delta n(z)$ such that,

$$\begin{aligned} d_1(z) &= d_0(z) + \Delta d(z) \\ n_1(z) &= n_0(z) - \Delta n(z), \end{aligned}$$

then

$$\begin{aligned} & n_0(z)d_1(z) - n_1(z)d_0(z) \\ &= n_0(z)\Delta d(z) + d_0(z)\Delta n(z) \\ &= S(z)(\theta_1 - \theta_0), \end{aligned} \tag{15}$$

where $S(z) = \begin{pmatrix} 1 & z^{-1} & \cdots & z^{-2n+1} & z^{-2n} \end{pmatrix} \times V$ and the matrix, V, is a Sylvester matrix of two polynomials $n_0(z)$ and $d_0(z)$. Therefore, from (14),

$$Q(z) = U_0^{-1}(z)S(z)(\theta_1 - \theta_0)U_1^{-1}(z). \tag{16}$$

Theorem 2 (Continuity in the δ_ν-Gap Metric Space): Parametrized controller, $C(\theta_i)$, is continuous in the δ_ν-gap in the metric space $(\mathscr{C}, \delta_\nu(\cdot,\cdot))$.

Proof: Given $\varepsilon > 0$, choose $\delta > 0$ such that,

$$\delta = \frac{\varepsilon}{||U_0^{-1}(z)S(z)U_1^{-1}(z)||_\infty}. \tag{17}$$

Then if $|\theta_0 - \theta_1| < \delta$ and $\theta_0,\ \theta_1 \in \Theta$,

$$\begin{aligned} \delta_\nu(C(\theta_0), C(\theta_1)) &\le ||T(P, C(\theta_0)) - T(P, C(\theta_1))||_\infty \\ &= ||Q(z)||_\infty \\ &\le |\theta_0 - \theta_1|\ ||U_0^{-1}(z)S(z)U_1^{-1}(z)||_\infty \\ &< \ \delta\ ||U_0^{-1}(z)S(z)U_1^{-1}(z)||_\infty \\ &= \ \varepsilon\ . \end{aligned} \tag{18}$$

Since we can make the size of ε as small as we want, the controller moves continuously at θ_0 in the parameter space Θ in the δ_ν-gap metric. ■

Now we show there exists a finite covering of a set of controllers, $\mathscr{C}$, by ε-balls. For every controller $C(\theta_\rho)$ contained in the ball $\mathscr{B}(C(\theta_i), \varepsilon)$, which is centered at $C(\theta_i)$ with the radius of ε, if we choose ε less than $b_{P,C_{\theta_i}}$, than $\delta_\nu(C(\theta_i), C(\theta_\rho)) < b_{P,C(\theta_i)}$. In addition to this, if $C(\theta_i)$ is certified, every controller $C(\theta_\rho)$ in the ball $\mathscr{B}(C(\theta_i), \varepsilon)$ guarantees to stabilize the unknown actual plant $P(z)$ by Theorem 1. Since the radius ε is strictly positive, we are able to construct a finite number of non-vanishing balls such that,

$$\mathscr{C} \subset \bigcup_{i=0}^{N} \mathscr{B}(C(\theta_i), \varepsilon), \tag{19}$$

where N is a finite number. Therefore by doing at least N times of experiment, the certification problem of $\mathscr{C}$ will be solved.

VI. Conclusions

We have developed a search method for a subset of designed controllers to reduce the number of experiments required in controller certification. By doing experiments only on a small subset of controllers, we can solve the certification problem for a large set of candidate controllers. Precomputed design quantities such as $b_{\hat{P},C}$ and $\delta_\nu(C_i, C_j)$ are used to guide the search for controllers to be tested experimentally with the actual plant to yield certification of the complete set $\mathscr{C}$. A required degree of performance for the controller certification can be maintained as we appropriately choose α in (3). When the candidate controller set has an infinite number of controllers, we showed that only a finite number of experiments are required to solve the certification problem.

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