# Adaptive Channel Modeling for MIMO Wireless Communications

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***Abstract*— The application of state-space-based subspace system identification methods to training-based estimation for time-varying MIMO frequency-selective channels is explored with the motivation of possible parsimonious parametrization and direct model complexity control. The comparison between the state-space-based channel estimation algorithm and the FIR-based RLS algorithm shows the former is a more robust modeling approach than the later.**

## I. Introduction

Digital communication using multiple transmit and receive antennas has been one of the most important technical developments in modern communications. In a rich scattering environment, multi-input-multi-output (MIMO) systems offer significant capacity gain at no cost of extra spectrum [1]. Furthermore, space-time channel codes [2] can be applied to utilize multiple transmit antennas to build spatial redundancy in the transmitted signal such that maximum spatial diversity is achieved. Coherent space-time processing schemes assume the availability of a channel model at the receiver. Therefore, this model needs to be estimated at the receiver end.

For slowly-varying channels, training-based channel estimation is very common in practice. Most channel estimation schemes assume an FIR channel model. FIR models for wireless channels have been widely used for their simplicity and guaranteed stability. An FIR model represents the sub-channels of a MIMO channel with *separately* parametrized finite-length impulse responses. However, when the spatial subchannels are correlated with each other due to insufficient separation between adjacent antennas, an FIR model can be very non-parsimonious and contain excessive redundancy. Therefore it may be beneficial to adopt a state-space model which treats the whole channel as a single entity and hence captures the structure in the channel while allowing a more parsimonious description of the MIMO channel.

It is the subject of this paper to explore the application of state-space models to represent MIMO frequency-selective channels in the hope for possible parsimonious parameterization. Since any channel model is essentially an approximation of the physical channel, the goal of channel modeling is to find a model that is as close to the real channel as possible and maintains manageable complexity as well. It is found that state-space models are able to provide low-order models of high-quality channel approximation.

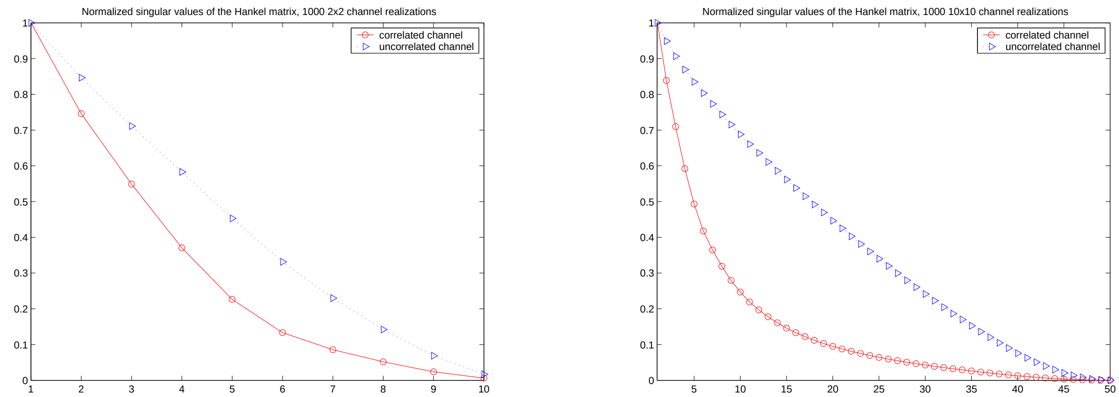
Low-order, high-quality models are of interest because they hold the prospect of requiring fewer parameters for their description and consequently an improvement in adaptation rate. Since we are considering learning with training data, the adaptation rate is important, because training symbols contain no new message information. The simulations of the paper demonstrate comparable (but slower) adaptation performance of state-space methods but improved approximation properties.

For the purpose of channel estimation, an algorithm for estimating MIMO state-space models is needed. Subspace system identification (SSI) algorithms are a group of methods that identify a MIMO state-space system in a straightforward way using numerically robust computational tools such as singular value decomposition (SVD) and QR factorization [3], [4]. SSI algorithms provide a direct way to control the complexity of the estimated channel model. The order of the channel model can be selected by the user by choosing the number of the largest singular values of the estimated extended observability matrix to include. Furthermore, the modeling error is assessable as it is related to the sum of the neglected Hankel singular values.

This paper is organized as follows. Section II describes the redundancy in correlated MIMO channels and the motivation for the adoption of state-space models. Section III reviews the subspace system identification methods and discusses its application to MIMO channel estimation. Section IV presents numerical simulation results and compares the state-space-based SSI channel estimation algorithm to the conventional FIR-based RLS channel estimation algorithm. The paper is concluded in Section V.

## II. Channel Redundancy and State-Space Channel Model

In order to understand the redundancy in spatially correlated MIMO channels and its effect on the order or the McMillan degree of the channel, a computer experiment was carried out to examine the singular values of the Hankel matrix of correlated MIMO channels and compare them with those of spatially white MIMO channels. It will be shown that channel




(a) $2 \times 2$ channels.

(b) $10 \times 10$ channels.

Fig. 1. Hankel singular values of MIMO channels (normalized with respect to the largest value). $\rho$ = 0.8 for the spatially correlated channels; $\rho = 0$ for spatially white channels.

correlation increases the number of negligible small Hankel singular values and hence reduces the effective channel order.

In the simulation, the MIMO channel is assumed to be composed of multiple finite impulse responses. Assuming all the subchannels have the same length $L$, an FIR MIMO channel with $m$ transmit antennas and $p$ receive antennas can be represented by assembling all the taps of the subchannel impulse responses with the same delay $\tau$ into a matrix $\mathbf{H}_\tau$ and represent the channel as a series of matrices

$$[\mathbf{H}_0, \mathbf{H}_1, \ldots, \mathbf{H}_{L-1}], \quad \mathbf{H}_\tau = \begin{bmatrix} h_{11}(\tau) & h_{12}(\tau) & \ldots & h_{1m}(\tau) \\ h_{21}(\tau) & h_{22}(\tau) & \ldots & h_{2m}(\tau) \\ & & \vdots & \\ h_{p1}(\tau) & h_{p2}(\tau) & \ldots & h_{pm}(\tau) \end{bmatrix} \tag{1}$$

The simulated channels are taken to be square symmetric, and the length of the impulse responses is set to $6$. The channel taps are generated as zero-mean circularly-symmetric complexity Gaussian (ZMCSCG) random variables. The impulse response of each subchannel is normalized to have unit energy. The MIMO channel is assumed to the uncorrelated in delay but correlated in spatial dimension, i.e.

$$E(\mathbf{vec}(\mathbf{H}_k)\mathbf{vec}(\mathbf{H}_l)^H) = \begin{cases} \mathbf{0}, k \neq l \\ \mathbf{R}, k = l \end{cases} \tag{2}$$

where $\mathbf{vec}(*)$ is an operator that stacks the columns of a matrix on top of each other to form a vector. We adopt a simple model for the spatial correlation structure of $\mathbf{H}_\tau$ [5]

$$\mathbf{H}_\tau = (R_{RX})^{1/2}\mathbf{H}_\mathbf{w}(R_{TX})^{T/2} \tag{3}$$

$$\mathbf{R} = R_{TX} \otimes R_{RX} \tag{4}$$

where $\mathbf{H}_\mathbf{w}$ is a $p \times m$ matrix with IID ZMCSCG elements. "$\otimes$" denotes the Kronecker product. $R_{TX}$ and $R_{RX}$ are, respectively, the transmit covariance matrix and receive covariance matrix. They are assumed to be the same in the simulation and possess the structure

$$R_{TX} = R_{RX} = \begin{bmatrix} 1 & \rho & \rho^2 & \ldots & \rho^{m-1} \\ \rho & 1 & \rho & \ldots & \rho^{m-2} \\ & & \vdots & & \\ \rho^{m-1} & \rho^{m-2} & \ldots & \rho & 1 \end{bmatrix} \tag{5}$$

where $\rho$ represents the correlation coefficient between the fading of two adjacent transmit or receive antennas. The value of $\rho$ is chosen to be 0.8 in the simulation.

As an indicator of the effective channel order, the singular values of the Hankel matrix of the corresponding MIMO channels are computed and averaged over 1000 channel realizations. [The $\mathrm{H}_\infty$-norm of the difference between a MIMO system and its low-order approximation is related to the sum of the neglected Hankel singular values, see [6] (Theorem 7.11).] Figure 1 shows the difference between the distributions of Hankel singular values for spatially white channels and spatially correlated channels. It can be seen that spatial correlation increases the number of negligible singular values and hence allows a low-order state-space model that is a high-quality approximation of the original full-order channel. Furthermore, the larger the size of the MIMO channel is, the lower the order of a high-quality channel model can be achieved. Therefore, a state-space model is capable of providing good low-order approximations of large spatially correlated MIMO channels.

$$\begin{aligned} \mathbf{x}_{k+1} &= A\mathbf{x}_k + B\mathbf{u}_k \\ \mathbf{y}_k &= C\mathbf{x}_k + D\mathbf{u}_k + \mathbf{n}_k \end{aligned} \tag{6}$$

where
$\mathbf{u}_k$ – $m \times 1$ input vector at time $k$;
$\mathbf{y}_k$ – $p \times 1$ output vector at time $k$;
$\mathbf{x}_k$ – $n \times 1$ state vector at time $k$;
$\mathbf{n}_k$ – $p \times 1$ white Gaussian noise vector at time $k$.

On the other hand, the order of an FIR channel can only be controlled by the length of the impulse response. This is an inferior approach compared to the model reduction in state-space models, as will be shown later in Section IV.

## III. Subspace System Identification and MIMO Channel Estimation

The MIMO FIR model can be fitted by forming a parameter vector from the columns of $\mathbf{H}_\tau$ and a regressor vector from a suitable rearrangement of the input signals at the transmit antennas. Then Recursive Least Squares is applied to this vector problem.

However, for state-space models, a special class of estimation algorithms is needed. Subspace system identification (SSI) refers to a class of recent algorithms, such as MOESP and N4SID, which apply input-output system identification methods to determine directly a state-space realization of a system. The key idea of SSI methods is to estimate the extended observability matrix through the projection of future input-output data onto past input-output data based on the relationship between Hankel matrices of the input and output given by

$$Y_{1,i,M} = \Gamma_i X_{1,M} + H_i U_{1,i,M} + N_{1,i,M},\ i > n. \tag{7}$$

where $U_{1,i,M}$, $Y_{1,i,M}$ and $N_{1,i,M}$ are Hankel matrices of the input, output and noise, respectively, with the form of

$$Y_{1,i,M} = \begin{bmatrix} \mathbf{y}_1 & \mathbf{y}_2 & \cdots & \mathbf{y}_M \\ \mathbf{y}_2 & \mathbf{y}_3 & \cdots & \mathbf{y}_{M+1} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{y}_i & \mathbf{y}_{i+1} & \cdots & \mathbf{y}_{i+M-1} \end{bmatrix}. \tag{8}$$

$X_{1,M}$ is a matrix containing the state vectors

$$X_{1,M} = \begin{bmatrix} \mathbf{x}_1 & \cdots & \mathbf{x}_M. \end{bmatrix}$$

$\Gamma_i$ and $H_i$ are, respectively, the extended observability matrix and the matrix of impulse response coefficients.

$$\Gamma_i = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{i-2} \\ CA^{i-1} \end{bmatrix}, H_i = \begin{bmatrix} D & 0 & & & \cdots & 0 \\ CB & D & 0 & & \cdots & 0 \\ CAB & CB & D & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & & \vdots \\ CA^{i-2}B & & & & \cdots & D \end{bmatrix}. \tag{9}$$

Then the system matrices $A$, $B$, $C$ and $D$ are computed based on the estimated observability matrix and singular value decomposition (SVD) algorithm, see [3], [4], [7] for in-depth treatments.

Based on the channel model given in (6), SSI methods require the input to satisfy the following requirements for the channel to be identifiable.

1) The input $\mathbf{u}_k$ is uncorrelated with the additive Gaussian white noise $\mathbf{n}_k$.
2) The input $\mathbf{u}_k$ is persistently exciting of order of at least 2 times the maximum order of the channel.
3) The symbols in the input sequence are contiguous and, for consistency, the number of inputs goes to infinity.

The first assumption is usually satisfied for wireless communication systems. The second one requires the training sequence to maintain a certain structure. Notice that the third assumption places limitation on the application of SSI methods to channel estimation in wireless communication systems where the training sequences are usually not contiguous in time.

In this paper, we consider a special type of SSI algorithms, the MOESP algorithm [8] and its recursive version [9]. Detailed discussion of SSI for non-contiguous data and the recursive MOESP for time-varying channels is given in APPENDIX.

## IV. Simulation and Interpretation

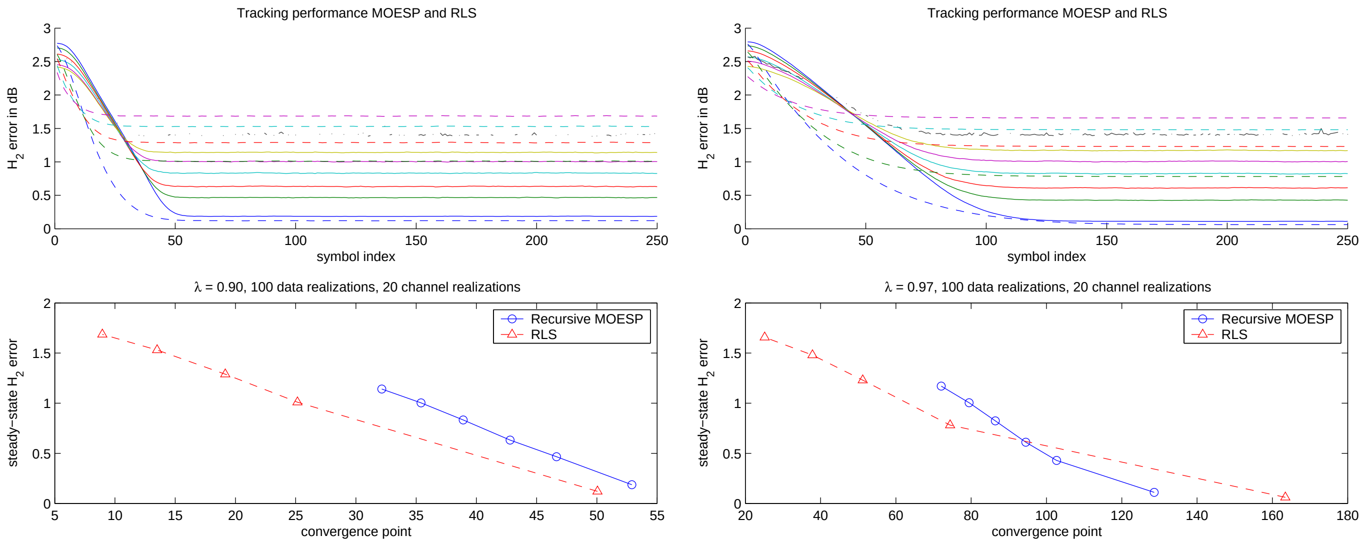
The purpose of the simulation study is to examine the estimation performance of state-space-based recursive SSI channel estimation algorithm and to compare it to the FIR-based recursive least squares (RLS) channel estimator.

The spatially correlated MIMO channels are generated in the same way as discussed in Section II. The training sequences are random binary sequences that are uncorrelated in both spatial dimension and time dimension. We choose the $H_2$ norm of the difference between the estimated channel model and the true channel as an indicator of the estimation error. For a fixed MIMO FIR-6 channel with temporally and spatially white input symbol sequence, both the SSI and FIR/RLS channel estimators were run until they reached a stationary value. Then, at time $t = 0$, the channel was changed and the identification methods allowed to reconverge. This is depicted in Figure 2(a) and 2(b) for $2 \times 2$ channels with SNR=20dB and illustrates the transient performance or learning rate of the identifiers.

For the upper figures, the horizontal axis shows the number of training symbols sent after time $t = 0$. The vertical axis shows the $H_2$ error between the true channel and the model. There are separate curves for each different model order, with the dashed curves corresponding to the FIR models and the solid or dotted curves corresponding to state-space models. The items of interest are the steady-state error values, which indicate the modeling accuracy, and the learning or adaptation rate. The ordering of the curves shows that lower-order models achieve poorer steady-state accuracy and faster learning. The curves themselves are the result of averaged curves over 20 channel (initial and final) realizations and, for each channel realization, 100 training symbol sequence realizations.

The lower figures are derived from the upper figures and plot one point for each model order and identification method, with triangles for the FIR/RLS method and circles for SSI/MOESP. The vertical axis is the steady–state modeling error and the horizontal axis shows the input symbol number from which the channel model error remains below 110% of the steady-state value. This latter figure gives a measure of convergence rate.

The state-space models provide low-order models of high-quality approximation, especially in the presence of channel redundancy, whereas the reduced-length FIR models fail to provide comparable estimation error performance. This performance difference between reduced-order state-space models




(a) $\lambda = 0.90$, fast convergence.

(b) $\lambda = 0.97$, slow convergence.

Fig. 2. Estimation error of MOESP and RLS for $2 \times 2$ spatially correlated channels with $\rho$ = 0.8.

and reduced-length FIR models becomes even more evident in large-dimension MIMO channels (Figure 3). The reason for such a difference is that state-space models are a more general model class than FIR models. The former includes the latter as a special case and thus provides a larger model set. Therefore, when the true order or length of the physical channel is unknown, the state-space model is a more robust channel modeling approach than the FIR model because the former is less sensitive to model order selection error than the latter.

The convergence rate of the recursive MOESP algorithm is in general slower than that of the RLS algorithm, but the difference stays in a comparable range. It is also observed that the recursive MOESP algorithm exhibits a start-up delay which is not present for the RLS algorithm. We suspect that this delay can be remedied by careful modification of the MOESP algorithm.

The faster convergence of RLS is to be expected because the (spatially and temporally) white training sequence is optimal for FIR model structure [7]. However, for the ultimate application of these models for the equalization of the MIMO channel, where the models themselves might be used to capture FIR or state-space approximations to the channel inverse, it is not immediately apparent which method would be faster. One suspects that, since MOESP includes an RLS section to produce the $B$ and $D$ matrices of the state-variable realization, that RLS would be always faster than SSI of the same order.

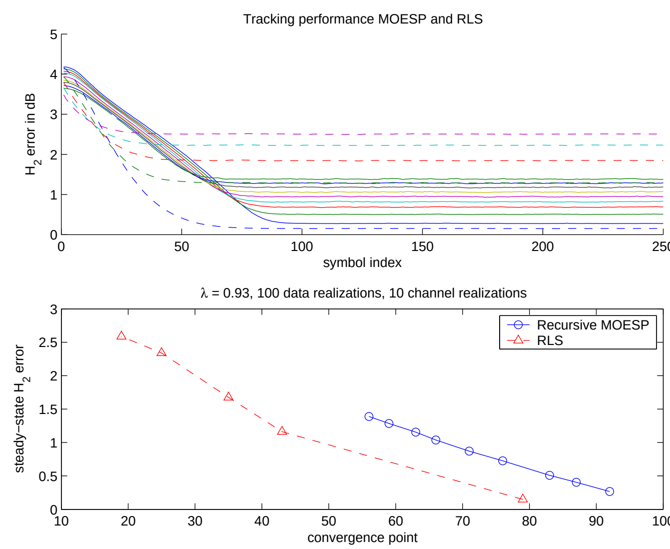



Fig. 3. Estimation error of MOESP and RLS for $3x3$ spatially correlated channels with $\rho = 0.8$, $\lambda = 0.93$.

## V. Conclusion

In this paper, the state-space model is proposed for modeling MIMO wireless channels with the motivation for a more parsimonious parameterization. A recursive subspace system identification (SSI) algorithm for non-contiguous training data is presented for channel estimation in multiple-antenna communication systems. The comparison between the state-space-based channel estimation and the FIR-based RLS algorithm also shows the former is a more robust modeling approach than the later. Future work may involve improvements of the recursive SSI algorithms for faster convergence.

## Acknowledgment

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## APPENDIX

### A. SSI for non-contiguous data

Notice that the third assumption of conventional SSI algorithms (Section III) places limitations on the application of SSI methods to channel estimation in wireless communication systems where the training sequences are usually not contiguous in time. Instead, they lie in the overhead or mid-amble of a frame and are separated by data symbols. In this circumstance, the state evolution of the received data must be restarted at the frame boundaries. This is at variance with the standard formulation of SSI.

A suitable modification of the SSI algorithms for non-contiguous training data is proposed in [10]. It is shown that the non-contiguous-data approach is similar in estimation power to the contiguous-data approach when the length of the training sequence is sufficiently larger than the dimension of the extended observability matrix $\Gamma_i$ (9).

### B. Recursive SSI for time-varying channels

When the channel is time-varying, channel estimation needs to be carried out in a recursive manner so that any newly received data can be used to produce an up-to-date estimate of the channel. In this paper we mainly consider the application of a recursive version of the ordinary MOESP SSI scheme proposed in [9], whose key idea is to update the estimate of the extended observability matrix with the newly received data using LQ factorization. An exponential forgetting factor can also be added to remove the effect of data from the distant past at a high rate.

To give a brief summary of the recursive ordinary MOESP algorithm, consider that at time $t$ we have an LQ decomposition

$$\begin{bmatrix} U(t) \\ Y(t) \end{bmatrix} = \begin{bmatrix} R_{11}(t)_{mi\times mi} & 0 \\ R_{21}(t)_{pi\times mi} & R_{22}(t)_{pi\times pi} \end{bmatrix} Q$$

where $U(t)$ and $Y(t)$ are respectively the Hankel matrices of the input and output data at time $t$ with the form

$$Y(t) = \begin{bmatrix} \mathbf{y}_0 & \mathbf{y}_1 & \cdots & \mathbf{y}_{t-i+1} \\ \mathbf{y}_1 & \mathbf{y}_2 & \cdots & \mathbf{y}_{t-i} \\ \vdots & \vdots & & \vdots \\ \mathbf{y}_{i-1} & \mathbf{y}_i & \cdots & \mathbf{y}_t \end{bmatrix}$$

It was shown in [3] that $R_{22}(t)$ is an estimate of the extended observability matrix at time $t$, $\Gamma_i(t)$, in the sense that they share the same column space.

The task of a recursive MOESP scheme would be to compute $R_{22}(t+1)$, an estimate of $\Gamma_i(t+1)$, based on the LQ factorization at time $t$ and the newly received data at time $t+1$. Suppose at time $t+1$ a new set of input-output data, $\mathbf{x}_{t+1}$ and $\mathbf{y}_{t+1}$, becomes available. We can form the following data vectors

$$\phi_u(t+1) = \begin{bmatrix} \mathbf{u}_{t-i+2} \\ \vdots \\ \mathbf{u}_t \\ \mathbf{u}_{t+1} \end{bmatrix}, \phi_y(t+1) = \begin{bmatrix} \mathbf{y}_{t-i+2} \\ \vdots \\ \mathbf{y}_t \\ \mathbf{y}_{t+1} \end{bmatrix}$$

Then the new input-output Hankel data matrix at time $k+1$ can be formed by appending the above vectors to the end of the Hankel data matrix at time $t$.

$$\begin{bmatrix} U(t+1) \\ Y(t+1) \end{bmatrix} = \begin{bmatrix} U(t) & \phi_u(t+1) \\ Y(t) & \phi_y(t+1) \end{bmatrix} \tag{10}$$

Without loss of generality, we only consider the case of (10). In order to find out $R_{22}(t+1)$, we rewrite (10) as

$$\begin{aligned} \begin{bmatrix} U(t+1) \\ Y(t+1) \end{bmatrix} &= \begin{bmatrix} \lambda U(t) & \phi_u(t+1) \\ \lambda Y(t) & \phi_y(t+1) \end{bmatrix} \\ &= \begin{bmatrix} \lambda R_{11}(t) & 0 & \phi_u(t+1) \\ \lambda R_{21}(t) & \lambda R_{22}(t) & \phi_y(t+1) \end{bmatrix} \begin{bmatrix} Q & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} R_{11}(t+1) & 0_{mi\times pi} & 0_{mi\times 1} \\ R_{21}(t+1) & \lambda R_{22}(t) & \phi(t+1) \end{bmatrix} Q' \end{aligned} \tag{11}$$

where the $pi \times 1$ vector $\phi(t+1)$ is obtained through a series of Givens rotation on the large lower triangular matrix. $\lambda \in (0,1]$ is an exponential forgetting factor which could be used to adjust the effect of the past input-output data on the estimate of the current channel model. $R_{22}(t+1)$ can be found by taking an LQ factorization of $[\lambda R_{22}(t)\ \phi(t+1)]$ as follows.

$$[\lambda R_{22}(t)\ \phi(t+1)] = R_0 Q_0 \tag{12}$$

Therefore, by substituting (12) into (11), we obtain

$$\begin{aligned} \begin{bmatrix} U(t+1) \\ Y(t+1) \end{bmatrix} &= \begin{bmatrix} R_{11}(t+1) & 0 & 0 \\ R_{21}(t+1) & R_0 Q_0 \end{bmatrix} Q' \\ &= \begin{bmatrix} R_{11}(t+1) & 0 & 0 \\ R_{21}(t+1) & R_0 Q_0 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} \\ &= \begin{bmatrix} R_{11}(t+1) & 0 \\ R_{21}(t+1) & R_0 \end{bmatrix} Q'' \end{aligned}$$

where $Q_1$ is a matrix that contains the first $mi$ rows of $Q'$ and $Q_2$ is a matrix that contains the last $pi+1$ rows of $Q'$. It is clear that

$$Q'' = \begin{bmatrix} Q_1 \\ Q_0 Q_2 \end{bmatrix}$$

is a matrix with orthogonal rows, i.e. $Q''(Q'')^T = I$. Thus

$$R_{22}(t+1) = R_0$$

In summary, the update from $R_{22}(t)$ to $R_{22}(t+1)$ is a two-step procedure: computing $\phi(t+1)$ in (11) using Givens rotation followed by computing $R_0$ in (12).

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