

Coordinated Control and Information Architecture

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Abstract—Coordinated control of vehicle formations is the motivating problem. The key idea is the use of Model Predictive Control (MPC) as a localized control law to connect the overall formation control performance and the inter-vehicle communication quality. The working problem is a simple 1-D vehicle formation with noisy channels. The measure of information quality is the covariance of the state estimates. A special form of MPC is used, which absorbs the estimate covariance into its probabilistically posed no-collision constraints. The resulting control law is deterministic and is adapted to the information quality.

I. INTRODUCTION

In recent years, coordinated control of vehicle formation has become a significant topic in control. Many studies imply that this area needs fundamental developments in control theories and new tool sets. The purpose of this paper is to explore the interaction between the control performance and the information quality of different information flow architectures. The working problem is a simple 1-dimensional vehicle formation task obeying a no-collision condition with noisy inter-vehicle communication.

Information flow structure is a central issue of coordinated formation control. Since in large scale vehicle formation problem no individual vehicle has the access to the global information, centralized control strategies are not applicable. Hence using a distributed control law with certain inter-vehicle communication is the main track. Studies on how information structure affects the stability have been made. In their formulation of [2], the graph Laplacian of the information structure is connected to the stability of the formation and a Nyquist-like stability criterion is stated. All the interchanged data are assumed to be accurate. In this paper, the information architectures of interest are built on limited communication capacity. All types of inaccuracy of communication (e.g. quantization error, ambient disturbances etc.) are modelled as additive white noises. Each vehicle uses the covariance of the estimate of others' positions as the measure of its available information quality. This enables the study of how information quality affects the control performance, and furthermore, enables the study of what are the minimum requirements on the information structure to achieve a specific performance quality. The sophistication of the control task is dependent on the quality of the communication.

With limited and imperfect knowledge of the formation, each vehicle should apply a localized control law that

accommodates the quality of its knowledge about others. Model Predictive Control (MPC) is an appropriate tool for this problem. Because it breaks a traditional multi-objective control task down to a single, online optimization task with multiple naturally posed constraints. Moreover, MPC controllers are reconfigurable, i.e. the formation task can be changed in flight via revising the optimization objective and the constraints. Due to the existence of random noises in our model, the constraints are posed in probabilistic form. Stochastic programming routines can be used to solve this probabilistically constrained MPC problem, as in [4]. There the problem is transformed into an equivalent nonlinear programming (NLP) problem which can be solved by a standard NLP solver. The method developed in [1] is adopted in this paper since only a deterministic Quadratic Programming solver is used. Moreover it uses the estimate covariance to change the probabilistic constraints. The resulting MPC control law accommodates the inaccuracy in estimates via the constraints. The control performance is thus adapted to the information quality. Application of MPC to coordinated multi-vehicle formations already exists, as in [3], where the application is focusing on how to stabilize the formation to a set of permissible equilibria.

In Section 2, two information structures and its corresponding MPC controllers are formulated. The simulation results are given in Section 3, which shows how the performance responds to the available information.

Notations: The number i in a superscript denotes that this variable is of the i th vehicle. And the subscripts stand for the time. $E_{\mathcal{I}_n^i}(\cdot)$ represents a conditional expectation $E(\cdot|\mathcal{I}_n^i)$ and $P_{\mathcal{I}_n^i}(\cdot)$ is a conditional probability $P(\cdot|\mathcal{I}_n^i)$, where \mathcal{I}_n^i is the set of the information collected by vehicle v_i up to time n . In some subscripts $[n+k|n]$ may be seen, such a quantity represents a certain state estimate or estimate covariance conditioning on the corresponding \mathcal{I}_n^i set.

II. WORKING PROBLEM DESCRIPTION

A. 1-D Coordinated Formation Scenario

Consider a simple 1-D coordinated formation control problem as the prototype coordinated flight problem. The formation consists of 3 mobile beads (vehicles) v_1, v_2 and v_3 on a wire with limited communication between them. Each bead's dynamic model is simply a controlled integrator with

certain noise:

$$x_{n+1}^i = x_n^i + u_n^i + \omega_n^i, \quad i = 1, 2, 3. \quad (1)$$

Here ω_n^i is a gaussian white noise with $\omega_n^i \sim N(0, q_n^i)$. The formation task is that the leader v_1 tracks a known reference trajectory, the followers v_2 and v_3 try to minimize the distance to the bead in front while obeying a no-collision condition. The followers are informed of the task (other beads' states, etc.) through noisy inter-vehicle communication.

This is a coordinated control task with distributed control law and information architecture. The overall performance is tightly bound to the quality of data communication. Hence, the local controllers should be adapted to the information quality and, indeed, the control task should determine the assigned bandwidth. Intuitively the control law should drive the beads to stay apart in order to guarantee no-collision when the knowledge of others' positions is coarse.

Since the task has been specified as an optimal control problem with constraints, Model Predictive Control (MPC) is an appropriate tool. The control diagram of each vehicle are identical as shown in Figure 1. Due to the underlying stochastic nature of our problem, the MPC problem to be solved by each bead is stated as a conditional expectation. Here we pose the MPC for bead v_1 first:

$$\min_{u^1} J_1 = \min_{u^1} E_{\mathcal{I}_n^1} \sum_{k=1}^N \left[(x_{n+k}^1 - r_{n+k})^2 + \lambda (u_{n+k-1}^1)^2 \right] \quad (2)$$

$$\text{subject to: } u_{n+k}^1 \in U,$$

where U is the admissible set for control variable u , r_{n+k} denotes the reference signal for the vehicle v_1 , \mathcal{I}_n^1 is the set of information collected by v_1 up to time n and $E_{\mathcal{I}_n^1}(\cdot)$ is the conditional expectation conditioning on that set. This is a constrained LQG problem.

Vehicles v_2 and v_3 should avoid collision with the previous bead. Due to the noises these constraints are of probabilistic form

$$P_{\mathcal{I}_n^i}(x_{n+k}^i \leq x_{n+k}^{i-1}) > p, \quad i = 2 \text{ or } 3, \quad (3)$$

where x^{i-1} is the previous vehicle's position, p represents some large probability level and $P_{\mathcal{I}_n^i}(\cdot)$ represents the conditional probability conditioning on \mathcal{I}_n^i .

In the next subsection, we will state two simple information structures and their corresponding localized and modified MPC control laws.

B. Information Architecture

Different information structures will give different impact on the control performance. In this subsection, we formulate two simple information structures. These suffice to convey our basic idea and will be a basis for further study.

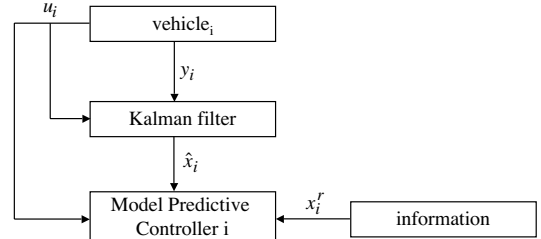


Fig. 1. Control block diagram of each vehicle

1) *Case 1*: The first information structure to explore is where each vehicle knows its own state perfectly and transmits its current state and its MPC control sequence to the next vehicle through a certain noisy channel. The follower estimates its own reference signal (i.e. the previous bead's position) for implementing the optimization procedure. Bead v_1 's MPC problem has been stated in (2) and there is no probabilistic constraint. The MPC problems for v_2 and v_3 need some special treatment. We state the MPC problem for v_2 first. Vehicle v_3 's MPC is similar to v_2 's with x^2 providing the reference and the collision constraint in place of x^1 .

At time n , the bead v_1 sends its current state x_n^1 and the optimizing control sequence $u_{n \dots n+N-1}^1$ to v_2 . The information about v_1 transmitted to v_2 is modelled as:

$$z_n^1 = x_n^1 + \nu_n^1, \quad (4)$$

$$\hat{u}_{n+k|n}^1 = u_{n+k|n}^1 + \eta_{n+k,n}^1, \quad (5)$$

where $\nu_n^1 \sim N(0, R_n^1)$ and $\eta_{n+k,n}^1 \sim N(0, s_{n+k,n}^1)$. And the finite horizon optimization problem to be solved by v_2 is

$$\min_{u^2} J_2 = \min_{u^2} E_{\mathcal{I}_n^2} \left\{ \sum_{k=1}^N \left[(x_{n+k}^2 - x_{n+k}^1)^2 + \lambda (u_{n+k-1}^2)^2 \right] \right\} \quad (6)$$

$$\text{subject to: } P_{\mathcal{I}_n^2}(x_{n+k}^2 \leq x_{n+k}^1) > p, \\ u_{n+k}^2 \in U.$$

Here x_{n+k}^1 plays the role of the reference trajectory for v_2 . For v_2 , solving (6) requires applying stochastic programming approach. We are going to use a modified version of MPC instead. This approach treats the information quality in terms of estimates' covariances nicely.

First, consider all the information that v_2 might have:

- 1) Perturbed position of v_1 : \hat{x}_n^1 ; v_2 needs a Kalman filter to give the best estimate of x_n^1 . For this specific vehicle

dynamics model, the Kalman filter is

$$\begin{aligned}\hat{x}_{n|n}^1 &= \hat{x}_{n|n-1}^1 + \frac{\Sigma_{n|n-1}^1(z_n^1 - \hat{x}_{n|n-1}^1)}{(\Sigma_{n|n-1}^1 - R_n^1)}, \\ \hat{x}_{n+1|n}^1 &= \hat{x}_{n|n}^1 + \hat{u}_{n|n}^1, \\ \Sigma_{n|n}^1 &= \Sigma_{n|n-1}^1 - \frac{(\Sigma_{n|n-1}^1)^2}{(\Sigma_{n|n-1}^1 + R_n^1)}, \\ \Sigma_{n+1|n}^1 &= \Sigma_{n|n}^1 + q_n^1 + s_{n,n}^1.\end{aligned}\quad (7)$$

- 2) Open-loop prediction of state x^1 and x^2 through the entire optimization horizon based on the measurements at time n . The open-loop predictor for x^1 is:

$$\bar{x}_{n+k|n}^1 = \bar{x}_{n+k-1|n}^1 + \hat{u}_{n+k-1|n}^1, \quad (8)$$

$$\sigma_{n+k|n}^1 = \sigma_{n+k-1|n}^1 + q_{n+k-1}^1 + s_{n+k-1,n}^1, \quad (9)$$

where $\bar{x}_{n+k|n}^1$ is the open-loop prediction with $\hat{x}_{n|n}^1 = \bar{x}_{n|n}^1$ provided by the Kalman filter, and $\sigma_{n+k|n}^1$ is the corresponding covariance with $\sigma_{n|n}^1 = \Sigma_{n|n}^1$. The open-loop predictor for v_2 's own states is

$$\bar{x}_{n+k|n}^2 = \bar{x}_{n+k-1|n}^2 + u_{n+k-1}^2, \quad (10)$$

$$\sigma_{n+k|n}^2 = \sigma_{n+k-1|n}^2 + q_{n+k-1}^2, \quad (11)$$

where $\bar{x}_{n+k|n}^2$ is the open-loop prediction with $\bar{x}_{n|n}^2 = x_n^2$, as we have assumed that each bead measures its own state perfectly, and $\sigma_{n+k|n}^2$ is the corresponding covariance with $\sigma_{n|n}^2 = 0$.

With all this information at hand, now we can modify the stochastic MPC problem (6) into a deterministic form by the following steps:

- 1) The criterion described in (6) is an expectation conditioned on all the information collected up to time n . Then we can express this expectation in a deterministic form in terms of its conditional means and covariances

as in [1].

$$\begin{aligned}J_2 &= E_{\mathcal{T}_n^2} \left\{ \sum_{k=1}^N [(x_{n+k}^2 - x_{n+k}^1)^2 + \lambda(u_{n+k-1}^2)^2] \right\} \\ &= \sum_{k=1}^N E_{\mathcal{T}_n^2} \left[(x_{n+k}^2)^2 - 2x_{n+k}^2 x_{n+k}^1 \right. \\ &\quad \left. + (x_{n+k}^1)^2 \right] + \sum_{k=1}^N \lambda(u_{n+k-1}^2)^2 \\ &= \sum_{k=1}^N \left[\sigma_{n+k|n}^2 + (\bar{x}_{n+k|n}^2)^2 - 2E_{\mathcal{T}_n^2}(x_{n+k}^2 x_{n+k}^1) \right. \\ &\quad \left. + \sigma_{n+k|n}^1 + (\bar{x}_{n+k|n}^1)^2 \right] + \sum_{k=1}^N \lambda(u_{n+k-1}^2)^2 \\ &= \sum_{k=1}^N (\sigma_{n+k|n}^2 + \sigma_{n+k|n}^1) + \sum_{k=1}^N \left\{ (\bar{x}_{n+k|n}^2)^2 \right. \\ &\quad \left. - 2E_{\mathcal{T}_n^2}[(x_n^2 + \sum_{k=1}^N u_{n+j}^2)(x_n^1 + \sum_{k=1}^N u_{n+j}^1)] \right. \\ &\quad \left. + (\bar{x}_{n+k|n}^1)^2 \right\} + \sum_{k=1}^N \lambda(u_{n+k-1}^2)^2 \\ &= \sum_{k=1}^N (\sigma_{n+k|n}^2 + \sigma_{n+k|n}^1) + \sum_{k=1}^N (\bar{x}_{n+k|n}^2 - \bar{x}_{n+k|n}^1)^2 \\ &\quad + \sum_{k=1}^N \lambda(u_{n+k-1}^2)^2\end{aligned}$$

As the covariance terms do not depend on the free variable u^2 , minimizing J_2 is equivalent to minimizing

$$\bar{J}_2 = \sum_{k=1}^N \left[(\bar{x}_{n+k|n}^2 - \bar{x}_{n+k|n}^1)^2 + \lambda(u_{n+k-1}^2)^2 \right] \quad (12)$$

- 2) Consider the probabilistic constraints. Note that by representing x_{n+k}^2 and x_{n+k}^1 in terms of their open-loop prediction and prediction errors:

$$x_{n+k}^2 = \bar{x}_{n+k|n}^2 + \sum_{j=0}^{k-1} \omega_{n+j}^2,$$

$$x_{n+k}^1 = \bar{x}_{n+k|n}^1 + \hat{x}_{n|n}^1 + \sum_{j=0}^{k-1} \omega_{n+j}^1 + \sum_{j=0}^{k-1} \eta_{n+j}^1,$$

where $\hat{x}_{n|n}^1$ is the estimate error of the Kalman filter. Hence,

$$\begin{aligned}x_{n+k}^2 &\leq x_{n+k}^1 \Leftrightarrow \\ \bar{x}_{n+k|n}^2 - \bar{x}_{n+k|n}^1 &\leq \sum_{j=0}^{k-1} (\omega_{n+j}^1 + \eta_{n+j}^1 - \omega_{n+j}^2) + \hat{x}_{n|n}^1.\end{aligned}$$

Let $\xi_{n+j} = \frac{\sum_{j=0}^{k-1} (\omega_{n+j}^1 + \eta_{n+j}^1 - \omega_{n+j}^2) + \bar{x}_{n|n}^1}{\sqrt{\sigma_{n+k|n}^1 + \sigma_{n+k|n}^2}}$. By assuming that $\bar{x}_{n|n}^1$ has gaussian distribution $N(0, \Sigma_{n|n}^1)$ and all these 0 mean white noises are independent and noticing that $\sigma_{n+k|n}^1 + \sigma_{n+k|n}^2$ is the sum of all the noises' variance appeared in the numerator, we conclude that ξ_{n+j} has standard normal distribution $N(0, 1)$. Therefore, the probabilistic constraint is equivalent to

$$\begin{aligned} P_{\mathcal{I}_n^2} \left(\frac{\bar{x}_{n+k|n}^2 - \bar{x}_{n+k|n}^1}{\Delta_{n+k|n}^2} \leq \xi_{n+k} \right) &> p \\ \Leftrightarrow \frac{\bar{x}_{n+k|n}^2 - \bar{x}_{n+k|n}^1}{\Delta_{n+k|n}^2} &< -\xi_{n+k}^* \\ \Leftrightarrow \bar{x}_{n+k|n}^2 &< \bar{x}_{n+k|n}^1 - \xi_{n+k}^* \Delta_{n+k|n}^2, \end{aligned} \quad (13)$$

where $\Phi(\xi_{n+k}^*) = p$, $\Phi(\cdot)$ is the standard normal distribution function and $\Delta_{n+k|n}^2 = (\sigma_{n+k|n}^1 + \sigma_{n+k|n}^2)^{\frac{1}{2}}$.

- 3) Note that in (13), $\Delta_{n+k|n}^2$ is a monotone increasing quantity and is putting more stringent requirements on $\bar{x}_{n+k|n}^2$ as k increases. This will make feasibility of the optimization a problem. Even if there is a feasible solution, it must be very conservative. Recall that the main idea of receding horizon control is that because there will be a new measurement available soon, only the first element of the optimizing control sequence is applied to the real plant. Dual to this idea, we use the closed-loop covariance $\Delta_{n+1|n}^2 = \Sigma_{n+1|n}^1 + q_n^2$ to modify the constraints instead of the pessimistic open-loop ones $\Delta_{n+k|n}^2$, because we know there are new measurements coming at the next sample time.
- 4) Hence the stochastic MPC problem that v_2 should be solving can be recast as the following deterministic problem:

$$\begin{aligned} \min_{u^2} \bar{J}_2 &= \min_{u^2} \sum_{k=1}^N \left[(\bar{x}_{n+k|n}^2 - \bar{x}_{n+k|n}^1)^2 \right. \\ &\quad \left. + \lambda (u_{n+k-1}^2)^2 \right], \\ \text{subject to: } \bar{x}_{n+k|n}^1 &= \bar{x}_{n+k-1|n}^1 + \hat{u}_{n+k-1|n}^1, \\ \bar{x}_{n+k|n}^2 &= \bar{x}_{n+k-1|n}^2 + u_{n+k-1}^2, \\ \bar{x}_{n+k|n}^2 &< \bar{x}_{n+k|n}^1 - \xi_{n+k}^* \Delta_{n+1|n}^2, \\ u_{n+k}^2 &\in U. \end{aligned}$$

Remarks:

- This approach is regulating the mean value process to a proper level in order to minimize the distance between two beads and to achieve the no-collision condition with the designed probability.
- The quantity $\Delta_{n+1|n}^2$ in the constraints represents the quality of the available information and $\xi_{n+k}^* \Delta_{n+1|n}^2$ represents the necessary standoff distance of v_2 from v_1 . Therefore, the resulting control law from this MPC

controller has been designed to fit the information quality.

Bead v_3 solves a similar MPC problem by taking v_2 's trajectory as the reference to track. The simulation result is included in section 3.

2) *Case 2:* It is well-known that the unidirectional information structure in Case 1 cannot achieve string stability. This can be seen from the simulation result in Section 3 – the last bead jiggles much more than the 1st. The reason is that all the perturbations are accumulated and transmitted via the channels and the last bead is designed to track.

As vehicles v_2 and v_3 had too little global information in Case 1, the second information structure will provide them r_n , the reference signal of v_1 , in addition to the communications in Case 1. The MPC problem for v_1 is identical to that in Case 1. Beads v_2 and v_3 solve a different MPC problem, which minimizes their distance from the reference trajectory while obeying the no-collision condition. The new MPC for v_2 is

$$\begin{aligned} \min_{u^2} J_2 &= \min_{u^2} E_{\mathcal{I}_n^2} \left\{ \sum_{k=1}^N (x_{n+k}^2 - r_{n+k})^2 \right. \\ &\quad \left. + \lambda (u_{n+k-1}^2)^2 \right\} \\ \text{subject to: } P_{\mathcal{I}_n^2} (x_{n+k}^2 &\leq x_{n+k}^1) > p, \\ u_{n+k}^2 &\in U. \end{aligned} \quad (14)$$

By the same procedure in case 1, we can derive the new modified MPC problem for v_2 :

$$\begin{aligned} \min_{u^2} \bar{J}_2 &= \min_{u^2} \left\{ \sum_{k=1}^N (\bar{x}_{n+k|n}^2 - r_{n+k})^2 \right. \\ &\quad \left. + \lambda (u_{n+k-1}^2)^2 \right\} \\ \text{subject to: } \bar{x}_{n+k|n}^1 &= \bar{x}_{n+k-1|n}^1 + \hat{u}_{n+k-1|n}^1, \\ \bar{x}_{n+k|n}^2 &= \bar{x}_{n+k-1|n}^2 + u_{n+k-1}^2, \\ \bar{x}_{n+k|n}^2 &< \bar{x}_{n+k|n}^1 - \xi_{n+k}^* \Delta_{n+1|n}^2, \\ u_{n+k}^2 &\in U, \end{aligned}$$

where $\Delta_{n+1|n}^2$ represents the closed-loop covariance. The MPC for v_3 can be stated similarly. The simulation result will be represented in the next section.

III. SIMULATION

In this section, the simulation results of the two cases stated in Section 2 are given. The reference trajectory of the whole formation is a ramp signal. All the noises are set to be 0 mean and variance 1 gaussian white noises. The optimization horizon N is 5 steps, $\lambda = 0.0001$ and the probability $p = 0.999$. Initial positions of the three beads are set to 6, 4 and 2. The initial position estimate of v_1 by v_2 is 5 and that of v_2 by v_3 is 3. The simulation ran for 200 samples and the simulation results are shown for 80 samples.

A. Result of Case 1

Figure 2 shows the achieved performance of the deterministic MPC controller posed in Section 2 for the unidirectional information flow. The reference trajectory $r(n)$ is the solid straight line. The dashed line that jiggles around the $r(n)$ is the trajectory of v_1 . Bead v_2 's trajectory is shown by the dots, and v_3 's is shown by the dots and solid line.

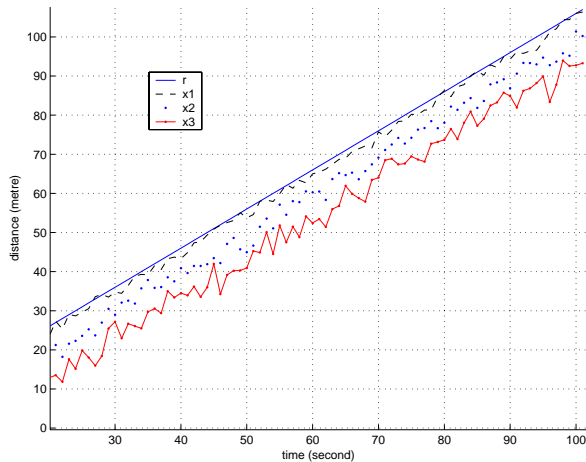


Fig. 2. Trajectories of the 3 vehicles with information structure Case 1

Remarks: It can be seen from the figure that no collision occurred during these 80 samples. Because the MPC has accommodated the state estimate's covariances into the control law, the resulting vehicle trajectory should represent the necessary standoff distance due to the inaccurate knowledge about other beads.

We remove the trends (best line fit) from the vehicles' trajectories and take the sample variance computed over 200 points of the residual data as an index of performance. The results are $\text{var}(v_1)=1.0685$, $\text{var}(v_2)=2.9850$ and $\text{var}(v_3)=5.4485$. The effect of the information architecture is evident in this increasing sample variance of the successive vehicles. As it also can be seen in Figure 2 that bead v_3 jiggles a lot more than v_1 . This is so-called string instability. Likewise, in this simple 1-D problem, the robustness of the entire formation is weak because the failure of a single bead or of a single communication link would destroy the completion of the task. Hence a different architecture is needed. But the MPC formulation and framework remain applicable.

B. Result of Case 2

As stated in Section 2, we inform v_2 to v_3 with the reference signal. The resulting trajectories are shown in Figure 3. The corresponding sample variances are $\text{var}(v_1)=0.9929$, $\text{var}(v_2)=2.4927$ and $\text{var}(v_3)=4.5365$. This result is slightly different from the previous case. The variance of v_1 is almost unchanged, since it is solving the same MPC problem. Vehicle v_2 and v_3 have smaller variances and their trajectories

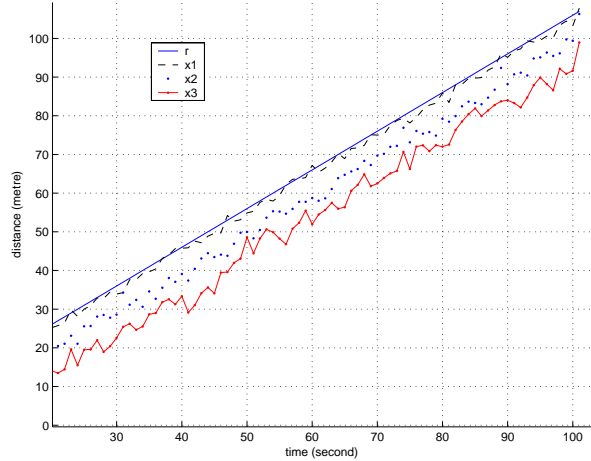


Fig. 3. Trajectories of the 3 vehicles with information structure Case 2

jiggled less than those in Case 1. This is the contribution of the additional information for v_2 and v_3 . However, string instability still happened, this needs further study.

IV. CONCLUSION

This paper describes the research of the interaction between the information quality in terms of estimate covariances and the control performance. The connection between these two aspects are made possible by a modified version of MPC controller, which can incorporate the information quality into the control law and keep the same framework while investigating different information structures with different quality. Two information architectures have been studied, string stability needs further consideration of communication strategy.

V. ACKNOWLEDGMENTS

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VI. REFERENCES

- [1] Jun Yan and Robert R. Bitmead, "Model Predictive Control and State Estimation: A Network Example", in *Proceedings of the 15th IFAC*, Barcelona, SPAIN, 2002, pp. 2381.
- [2] J. Alexander Fax and Richard R. Murray, "Graph Laplacians and Stabilization of Vehicle Formations", in *Proceedings of the 15th IFAC*, Barcelona, SPAIN, 2002, pp. 2353.
- [3] William B. Dunbar and Richard M. Murray, "Model Predictive Control of Coordinated Multi-Vehicle Formations", in *Proceedings of the 41st IEEE Conference on Decision and Control*, Las Vegas, Nevada, USA, 2002, pp. FrP08-4.
- [4] Pu Li and Moritz Wendt and Günter Wozny, "A Probabilistically Constrained Model Predictive Controller", *Automatica*, vol. 38, 2002, pp 1171-1176.