Lecture 9
Iterative control approaches

Modeling meets control meet modeling meets control meets modeling meets modeling ...
An archetype

Our overall aim is to minimize a performance function on the true system

\[ J_{\text{global}} = J(G_0, C) = \| \frac{WH_0}{1 + G_0 C} \|_{\infty} \]

Define “local” criterion

\[ J_{\text{local}} = \hat{J}(G_0, G_{\theta}, C) = \| \frac{(G_0 - G_{\theta}) C}{WH_0} \|_{\infty} \]

Look at the recursion

\[ C_i = \arg \min_C J_{\text{local}}(G_0, G_{\theta, i}, C) \]
\[ G_{\theta, i+1} = \arg \min_{\theta} J_{\text{local}}(G_0, G_{\theta}, C_i) \]

This is a descent procedure, where each modeling and control design phase drives down the value of the local criterion, which trades nominal performance against stability robustness.

This particular approach is infeasible because it requires knowledge of the true plant.

So we try an approximate approach using \( H_2 \) instead of \( H_\infty \)

Then the identification uses closed-loop data.
Hakvoort and the Dutch masters

Fix the control design method .... LQG or minimum variance

Adjust the identification noise model iteratively, and based on the current plant model, to create a control-relevant criterion via the noise model

\[ H_f = (1 + G\theta C) \]

Rinse and repeat

The aim is to prefilter the prediction error to yield the optimal data properties associated with identifying a model for subsequent LQG control design - a theory developed by Gevers and Ljung for the case of the system being in the model set. The optimal identification conditions occur when the system is optimally controlled.

This indirect approach works in examples but is not theoretically fully supported ... nor is any other scheme
The windsurfer scheme

An approach of increasing performance

Step 1: excite two closed loops, one with the real plant and the designed closed loop with the model. Determine satisfaction with modeling performance

happy: move to Step 2

sad: identify a new model using Hansen scheme based on coprime factor methods, control-relevant

Step 2: design a new feedback controller using Internal Model Control with larger bandwidth, go to Step 1

applicable for stable plants, closed-loop bandwidth is a parameter

Step 2A: use control-oriented model reduction as possible
Internal model control

[not to be confused with Internal Model Principle or Model Predictive Control] due to Morari and Zafiriou

Start with stable plant factored into an all-pass part and a minimum-phase part \( G_i = [G_i]^a[G_i]^m \)

Pick a desired closed-loop bandwidth \( \lambda_i \) and corresponding filter

\[
F_i = \left( \frac{\lambda_i}{s + \lambda_i} \right)^n
\]

Define \( Q_i = [G_i]^{-1}F_i \) (\( n \) chosen to keep \( Q_i \) proper)

and \( K_i = \frac{Q_i^{-1}F_i}{1 - G_iQ_i} \)

Then the designed closed loop is

\[
\frac{G_iK_i}{1 + G_iK_i} = F_i[G_i]^a
\]
Assume we have a stable closed loop with the real plant $G_0$.
Suppose we know another plant model $G$ which is stabilized by $C$.
Then we may write coprime factor descriptions of $G$ and $C$ as:

$$G = \frac{N}{D} \quad C = \frac{X}{Y}$$

where the factors are stable proper transfer function.
Without loss of generality we can assume $NX + DY = 1$.
Since $G_0$ is also stabilized by $C$, it can be written:

$$G_0 = \frac{N + RY}{D - RX}$$

for some stable proper transfer function $R$.
Let’s try to estimate $R$. 

---

Approximate System Identification & Control
Coprime factor identification

Known \( G = \frac{N}{D} \quad C = \frac{X}{Y} \)  
Find \( G_0 = \frac{N + RY}{D - RX} \)

Hansen, Franklin and Kosut noted the following

Create signals \( \alpha_t = X r_t \) and \( \beta_t = Dy_t - Nu_t \)

\[ \beta_t = R\alpha_t + (D - RX)He_t \]

[simple algebra yields this relation]

Now fit the Youla-Kucera parameter transfer function \( R_\theta \) and the noise model by minimizing the filtered prediction error

\[ \varepsilon_{\theta,t}^f = Y(\beta_t - R_\theta \alpha_t) \]

\[ = \left( \frac{G_0C}{1 - G_0C} - \frac{GC}{1 + GC} \right) r_t + \frac{1}{1 + G_0C}H_0e_t \]

Filtered reference and data signals yield independent \( (\alpha_t, \beta_t) \)

Freedom from bias, control-relevant
The Zang scheme

\[ J_{\text{global}}(G_0, C) = \lim_{N \to \infty} \frac{1}{N} \sum_{t=1}^{N} y_t^2 + \gamma u_t^2 = \left\| \gamma^{1/2} \frac{H_0}{1+G_0 C} H_0 \right\|_2 \]

Control design

\[ J_{\text{local}}(G_\theta, C) = \lim_{N \to \infty} \frac{1}{N} \sum_{t=1}^{N} y_t^2 + \gamma u_t^2 = \left\| \gamma^{1/2} \frac{H_\theta}{1+G_\theta C} H_\theta \right\|_2 \]

then use the frequency-weighting trick from last lecture

\[ J_{\text{local}}^F(G_\theta, C) = \lim_{N \to \infty} \frac{1}{N} \sum_{t=1}^{N} (Fy)_t^2 + \gamma (Fu)_t^2 = \left\| \gamma^{1/2} \frac{H_\theta F}{1+G_\theta C} H_\theta F \right\|_2 \]

Hopefully \[ J_{\text{local}}^F(G_\theta, C) \to J_{\text{global}}(G_0, C) \]

Intersperse with control-relevant modeling

\[ \left\| \gamma^{1/2} \frac{H_0}{1+G_0 C} H_0 \right\|_2 \leq \left\| \gamma^{1/2} \frac{H_\theta}{1+G_\theta C} H_\theta \right\|_2 + \left\| \left( \gamma^{1/2} \frac{H_0}{1+G_0 C} H_0 \right) - \left( \gamma^{1/2} \frac{H_\theta}{1+G_\theta C} H_\theta \right) \right\|_2 \]

based on minimizing the second term using closed-loop data and data filter

\[ |L_\theta|^2 = (1 + \gamma|C|^2) \left| \frac{H_\theta}{1 + G_\theta C} \right|^2 \]
References

   Chapters on Windsurfer & Zang schemes, control-relevant identification, sugar mill, CD player, wafer stepper applications


Zang, Bitmead & Gevers, “Iterative weighted least-squares identification and weighted LQG control design,” *Automatica*, vol. 31, pp. 1577-1594


Local time-line

1986 - Michel Gevers at the Australian National University
   Review of Clarke, Mohtadi & Tuffs GPC papers
1988 - Bob on sabbatical at University of Louvain, Belgium, visits to Van den Hof, Schrama, Hakvoort in Delft
   Draft of book “Adaptive Optimal Control: the Thinking Man’s GPC”
1989 - Started working with CSR Sugar at Victoria Mill
1990 - The George Polya moment
1990 - The Thinking Man’s book appears
1990/1 - Ari Partanen employed at Victoria Mill
   Train ride (pre-TGV) from Grenoble to Brussels with Michel Gevers
   20-page fax to Zhuquan Zang
   Discussions with Robert Kosut, co-editor of IEEE Trans AC special issue
1993 - Ari comes to ANU as a PhD student ... still has the passwords to CSR computers, experiments on the sugar mill
1993 - Raymond de Callafon, Paul Van den Hof and Okko Bosgra experiment on iterative identification and control on Philips
   CD player and ASML wafer stepper
1995 - Iterative Feedback Tuning ideas appear
1995 - Zang (theory) and Ari (application) papers appear in the same issue of Automatica
1999 - Pedro Albertos arranges a workshop in Valencia
   Leads to publication of the book “Iterative identification and control” 2002
1999 + - Lots of good work by Anderson, Gevers, Kosut, etc on cautious control